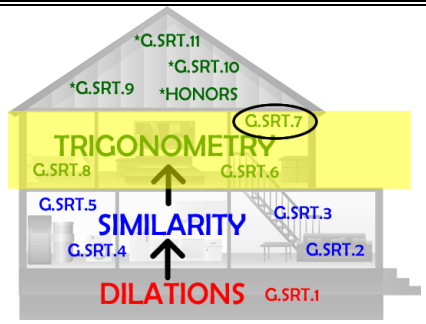
Objective	Interpretation of Objective	Connections
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.	This directly relates to the co-function relationship between these two ratios. The use of a trigonometric table would be essential here to see the pattern that $\sin(90-x)=\cos(x)$.	 This is not a critical concept for future work but it does represent a conceptual understanding of the ratios found in a triangle and so it is included here.

Student Outcomes	Emphasis	Tips
(1) The student will be able explain the co-function nature of sine and cosine. **(2)** The student will be able to calculate values that would make sine and cosine equal.	This is a very direct objective – establish the co-function relationship between Sine and Cosine.	1 – Use a trigonometry table to discover this relationship. Students easily find values that are equal in the Sine and Cosine columns. Then the natural questions to ask are why and when are these values equivalent?

CONCEPT 1 – Explain the relationship between the sine and cosine of complementary angles.

The trigonometry table can reveal a number of patterns that sometimes get hidden in calculator only use.

Deg	Sine (sin)	Cosine (cos)	Tangent (tan)		Deg	Sine (sin)	Cosine (cos)	Tangent (tan)
1	0.0175	0.9998	0.0175		46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349		47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524		48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699		49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875		50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051		51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228		52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405		53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584		54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763		55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944		56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126		57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309		58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493		59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679		60	0.8660	0.5000	1.7321
16	0.2756	0.9613	0.2867		61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057		62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249		63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443		64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640		65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839		66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040		67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245		68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452		69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663		70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877		71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095		72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317		73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543		74	0.9613	0.2756	3.4874
30	0.5000	0.8660	0.5774		75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009		76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249		77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494		78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745		79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002		80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265		81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536		82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813		83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098		84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391		85	0.9962	0.0872	11.430
41	0.6561	0.7547	0.8693		86	0.9976	0.0698	14.300
42	0.6691	0.7431	0.9004		87	0.9986	0.0523	19.081
43	0.6820	0.7314	0.9325		88	0.9994	0.0349	28.636
44	0.6947	0.7193	0.9657		89	0.9998	0.0175	57.290
45	0.7071	0.7071	1.0000		90	1.0000	0.0000	UND

PATTERN #1 – Identical Sine and Cosine Values

Look at the trigonometry table to the right, notice that every sine value has a matching value for cosine. Why would that happen?

Look at the pattern closer….

Sin 5° = Cos 85°
Sin 10° = Cos 80°
Sin 23° = Cos 67°
Sin 33° = Cos 57°
Sin 45° = Cos 45°

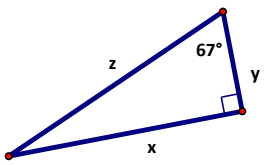
If the two angles are complementary then this relationship works:

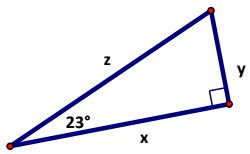
$$\sin\theta = \cos(90-\theta)$$

But why does that work?

The answer is quite a simple one. In a right triangle the two acute angles are always complementary and these two ratios are comparing the exact same sides of the triangle. Here's what that means:

Use the example of a 23°, 67° and 90° right triangle.





$$\sin\theta = \frac{Opposite}{Hypotenuse} \qquad \sin 67° = \frac{x}{z} \qquad \cos\theta = \frac{Adjacent}{Hypotenuse} \qquad \cos 23° = \frac{x}{z}$$

$$\cos\theta = \frac{Adjacent}{Hypotenuse} \qquad \cos 67° = \frac{y}{z} \qquad \sin\theta = \frac{Opposite}{Hypotenuse} \qquad \sin 23° = \frac{y}{z}$$

$$\sin 67° = \frac{x}{z} = \cos\ 23° \qquad \cos 67° = \frac{y}{z} = \sin 23°$$

The two complementary angles are in the same triangle and they are actually references the same sides.

1. Which of the following is equal to cos 35°

A) sin 35° B) cos 55° C) sin 55° D) cos 145°

2. Which of the following is equal to sin 8°

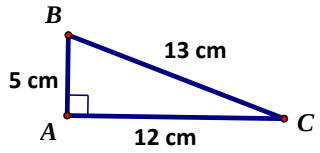
A) sin 82° B) cos 8° C) cos 82° D) sin 98°

3. Which of the following statements is false?

A) sin 45° = cos 45° B) sin 30° = cos 30° C) cos 10° = sin 80° D) sin 0° = cos 90°

4. Given the ratio $\frac{12}{13}$, which of the following is NOT equal to this value?

A) sin $\angle B$ B) cos $\angle C$ C) $\frac{AC}{BC}$ D) sin $\angle C$



5. If cos Θ = sin β then which of the following must be true?

A) $\Theta + \beta = 180°$ B) $\Theta - \beta = 90°$ C) $\beta = 90° - \Theta$ D) $\beta - \Theta = 90°$

6. Solve the following.

a) sin 27° = cos _____° b) cos 55° = sin _____° c) sin 17.8° = cos _____°

d) cos 90° = sin _____° e) cos 45° = sin _____° f) sin $62\frac{2}{3}°$ = cos _____°

7. Solve for the unknown.

a) sin (2x + 1°) = cos (22°) b) sin (5x + 15°) = cos (4x – 6°) c) sin (2x)= cos (x)

d) sin $(\frac{1}{2}x)$ = cos $(\frac{5}{2}x + 12)$ e) sin (7x + 15°) = cos (3x + 40°) f) sin $(\frac{1}{3}x + 2)$= cos (53°)

8. Explain WHY sin 20° = 0.342 and the cos 70° = 0.342.

Answers:

1) C

2) C

3) B

4) D

5) C

6) **a)** 63°

b) 35°

c) 72.2°

d) 0°

e) 45°

f) $27\frac{1}{3}°$

7) **a)** $x = 33.5$

b) $x = 9$

c) $x = 30$

d) $x = 26$

e) $x = 3.5$

f) $x = 105$

8) 20° and 70° are complementary which means that they are in the same right triangle. The sine of 20° compares the opposite and the hypotenuse but the cosine of 70° compares the adjacent and the hypotenuse but in this case the opposite of the 20° is the adjacent of the 70° because they are in the same triangle. Thus the two ratios are equal.